

1 Appendix: technical description of *CORTAX*

This appendix documents *CORTAX*. Section 1.2 derives from profit maximisation, the demand for labour, capital, location specific capital, intermediate inputs and financial assets for domestic and multinational firms. Taxes on corporate income, labour income, consumption and wealth are introduced when appropriate. The tax revenues have to meet the government expenditures on consumption, transfers and debt, see section 1.3. The market equilibria and the linkages with the Rest of the World are presented in section 1.4. Section 1.5 presents the solution procedure.

Notation follows some simple rules. Upper case symbols are used for aggregated values whereas lower case characters are reserved for per capita variables (in terms of the young generation in the country of origin). In the case of variables with two dimensions, the first index refers to the country which owns the resource (residence), whereas the second index denotes the using country (destination). Time subscripts and country indices are dropped in the exposition whenever this is possible.

The rates of return on bonds (\hat{r}_{wb}) and equities (\hat{r}_{we}) are assumed fixed. The considered countries are small in the sense that they can import (or export) capital from the Rest of the World (ROW) without affecting the world interest rates. In other words, the net supply of capital by the ROW is perfectly elastic. Multinationals are assumed to operate only in the other ‘small’ countries, but not in the ROW (and vice versa). The ROW block does not need to be fully modelled. International capital and good flows are restricted by the current account for each country.

1.1 Households

The overlapping generations framework follows the standard Diamond model (see Heijdra and Van der Ploeg (2002), Chapter 17). An individual is assumed to live for two periods: a working period and a retirement period. In deviation from the standard Diamond model, we assume that each period consist of T years. To keep the model tractable, we make a few simplifying assumptions. First, the consumption share in income is assumed constant when young and when old (i.e. within each period of T years). Since all income components grow at the annual rate g_a , consumption when young and when old grow at the same rate g_a . Second, young individuals supply the same amount of labour each year, independent of productivity growth. Old individuals do not work and thus have only non-labour income. In sum, households have to choose consumption paths $c_0^y(1+g_a)^s$ and $c_0^o(1+g_a)^s$ for the $s = 0, \dots, T - 1$ years in both periods.

Both young and old individuals hold assets in bonds and equities.

1.1.1 Population

The generation sizes are denoted by N^y and N^o , respectively. Total population $N = N^y + N^o$ might differ over countries but the population growth rate g_n is set identical for both countries since we focus on the steady state. This implies $N^y = (1 + g_n)^T N^o$. The relative population size is written as

$$\omega_n(i, j) \equiv N^y(i)/N^y(j) \quad (1.1)$$

1.1.2 Consumption and labour supply

Labour supply has to be a constant fraction of the time endowment. Therefore, we have to specify felicity v such that labour supply is constant even if productivity is growing. One option is to assume log-utility in consumption combined with a unit elasticity of intertemporal substitution, cf. Heijdra and Van der Ploeg (2002). A more flexible approach, which we will adopt here, is to assume that the value of leisure is growing at the productivity growth rate g_a .

$$v^y(\tau) = \begin{cases} \left[c^y(\tau)^{\frac{\sigma_l-1}{\sigma_l}} + \alpha_l (A_l(\tau)\hat{l}(\tau))^{\frac{\sigma_l-1}{\sigma_l}} \right]^{\frac{\sigma_l}{\sigma_l-1}} & \sigma_l \neq 1 \\ c^y(\tau)^{\frac{1}{1+\alpha_l}} (A_l(\tau)\hat{l}(\tau))^{\frac{\alpha_l}{1+\alpha_l}} & \sigma_l = 1 \end{cases} \quad (1.2)$$

where $c^{y,o}$ is consumption of goods, \hat{l} is leisure, $l = 1 - \hat{l}$ is labour supply, α_l is the weight of leisure and σ_l is the intratemporal substitution elasticity between consumption and leisure.¹ We assume that both consumption per capita and A_l grow at rate g_a . This implies that $v^y(\tau+1) = (1 + g_a)v^y(\tau)$. Equation (1.2) is combined with a similar expression for the ‘old’ generation, with the restriction that $\hat{l} = 1$, in:

$$\begin{aligned} U(t) &= \frac{1}{1-1/\sigma_u} \left[\sum_{\tau=0}^{T-1} \frac{v^y(t+\tau)^{1-1/\sigma_u}}{\rho_u^\tau} + \frac{\rho_o}{\rho_u^T} \sum_{\tau=0}^{T-1} \frac{v^o(t+T+\tau)^{1-1/\sigma_u}}{\rho_u^\tau} \right] \\ &= \frac{1}{1-1/\sigma_u} \left[v^y(t)^{1-1/\sigma_u} + \frac{\rho_o}{\rho_u^T} v^o(t+T)^{1-1/\sigma_u} \right] \sum_{\tau=0}^{T-1} \left(\frac{1+g_a}{\rho_u} \right)^\tau \end{aligned} \quad (1.3)$$

Wage income equals $w(1 - \tau_l)l$, where w denotes the gross wage rate, τ_l is the tax rate on labour and $\bar{w} = w(1 - \tau_l)$ the after tax wage rate. When young, total income, consisting of wage income $\bar{w}l$ and lumpsum transfers tr^y , is divided between consumption c^y and savings (net of interest income) s_n , see (1.4). Households of the old generations receive transfers tr^o , the pure profits accruing to location specific capital² π^o and they dissave, see (1.5). We abstract from bequests, such that households’ wealth equals zero at birth and death. Net savings for young households are:

$$s_n^y(t, t) = (1 - \tau_l)w(t)l + tr^y(t) - (1 + \tau_c)c^y(t) \quad (1.4)$$

¹ Sørensen (2001b) models labour supply differently by considering imperfect competition on the labour market. Unions with monopoly power set the wage rate and working hours by maximizing its members’ expected consumer surplus from work. Since the wage rate exceeds the market-clearing level, a fraction of the workers gets involuntary unemployed.

² We assume that location specific capital (a fixed factor) is owned by the old generation.

$$s_n^y(t, t + \tau) = (1 + g_a)^\tau s_n^y(t, t) \quad 0 \leq \tau < T$$

and similar for old households:

$$s_n^o(t, t + T) = \pi^o(t + T) + tr^o(t + T) - (1 + \tau_c)c^o(t + T) \quad (1.5)$$

$$s_n^o(t, t + T + \tau) = (1 + g_a)^\tau s_n^o(t, t + T) \quad 0 \leq \tau < T$$

Households accumulate wealth (assets a) according to:

$$\begin{aligned} a(t, t + \tau) &= \rho_s a(t, t + \tau - 1) + s_n^y(t, t + \tau), \quad 0 \leq \tau < T, \quad a(t, t - 1) = 0 \\ a(t, t + \tau) &= \rho_s a(t, t + \tau - 1) + s_n^o(t, t + \tau), \quad T \leq \tau < 2T, \quad a(t, t + 2T - 1) = 0 \end{aligned}$$

The wealth of the young generation accumulates as

$$\begin{aligned} a(t, t + \tau) &= s_n^y(t, t) \sum_{j=0}^{\tau} \rho_s^{\tau-j} (1 + g_a)^j, \quad \tau = 0, \dots, T - 1 \\ a(t, t + T - 1) &= s_n^y(t, t) \sum_{j=0}^{T-1} \rho_s^{T-1-j} (1 + g_a)^j = s_n^y(t, t) \frac{\rho_s^T - (1 + g_a)^T}{\rho_s - (1 + g_a)} \end{aligned} \quad (1.6)$$

Similarly, the wealth of the old generation decumulates as:

$$\begin{aligned} a(t, t + T + \tau) &= \rho_s a(t, t + T + \tau - 1) + s_n^o(t, t + T + \tau) \\ &= \rho_s^{\tau+1} a(t, t + T - 1) + s_n^o(t, t + T) \sum_{j=0}^{\tau} \rho_s^{\tau-j} (1 + g_a)^j, \quad \tau = 0, \dots, T - 1 \end{aligned}$$

For wealth in the final year (at age $2T$), this implies:

$$\begin{aligned} a(t, t + 2T - 1) &= \rho_s^T a(t, t + T - 1) + s_n^o(t, t + T) \sum_{j=0}^{T-1} \rho_s^{T-1-j} (1 + g_a)^j \\ &= \rho_s^T a(t, t + T - 1) + s_n^o(t, t + T) \frac{\rho_s^T - (1 + g_a)^T}{\rho_s - (1 + g_a)} \end{aligned} \quad (1.7)$$

Combine (1.6) and (1.7) with $a(t, t + 2T - 1) = 0$ and obtain the lifetime budget restriction:

$$\begin{aligned} s_n^y(t, t) &= -\frac{s_n^o(t, t + T)}{\rho_s^T} \\ \bar{w}(t)l + tr^y(t) - (1 + \tau_c)c^y(t) &= -\frac{(\pi^o(t + T) + tr^o(t + T) - (1 + \tau_c)c^o(t + T))}{\rho_s^T} \end{aligned} \quad (1.8)$$

Using the constant-growth assumption, we can write the budget equation for period t as:

$$\bar{w}(t)l + tr^y(t) - (1 + \tau_c)c^y(t) = -\left(\frac{1 + g_a}{\rho_s}\right)^T [\pi^o(t) + tr^o(t) - (1 + \tau_c)c^o(t)] \quad (1.9)$$

Maximizing (1.3) subject to (1.8) yields the first order conditions for c^y , \hat{l} and c^o , where λ_u denotes the Lagrange multiplier for the budget constraint:³

$$v^y(0)^{-1/\sigma_u} \left(\frac{v^y(0)}{c^y(0)}\right)^{1/\sigma_l} \sum_{s=0}^{T-1} \left(\frac{1 + g_a}{\rho_u}\right)^s = \lambda_u (1 + \tau_c) \quad (1.10)$$

³ Strictly speaking, a non-negative restriction on labour supply should be added. However, this restriction is normally not binding in this case.

$$\alpha_l A_l(0)^{1-1/\sigma_l} v^y(0)^{-1/\sigma_u} \left(\frac{v^y(0)}{\hat{l}(0)} \right)^{1/\sigma_l} \sum_{s=0}^{T-1} \left(\frac{1+g_a}{\rho_u} \right)^s = \lambda_u \bar{w}(0) \quad (1.11)$$

$$\frac{\rho_o}{\rho_u^T} v^o(T)^{-1/\sigma_u} \left(\frac{v^o(T)}{c^o(T)} \right)^{1/\sigma_l} \sum_{s=0}^{T-1} \left(\frac{1+g_a}{\rho_u} \right)^s = \frac{\lambda_u(1+\tau_c)}{\rho_s^T} \quad (1.12)$$

The first order conditions (1.10) and (1.11) imply that the marginal rate of substitution between consumption and leisure when young should equal the net wage rate:⁴

$$\hat{l} = \left(\frac{\alpha_l A_l(1+\tau_c)}{\bar{w}} \right)^{\sigma_l} \frac{c^y}{A_l} \quad (1.13)$$

The first and third equation imply the Euler equation:

$$\left(\frac{v^y(0)}{v^o(T)} \right)^{1/\sigma_l - 1/\sigma_u} \left(\frac{c^y(0)}{c^o(T)} \right)^{-1/\sigma_l} = \rho_o \left(\frac{\rho_s}{\rho_u} \right)^T$$

Use the assumption of steady state growth, meaning that both c and v grow at rate g_a , to rewrite this in terms of consumption in the first period as:

$$\left(\frac{v^y(0)}{v^o(0)} \right)^{1/\sigma_l - 1/\sigma_u} \left(\frac{c^y(0)}{c^o(0)} \right)^{-1/\sigma_l} = \rho_o \left(\frac{\rho_s}{\rho_u(1+g_a)^{1/\sigma_u}} \right)^T \quad (1.14)$$

1.1.3 Portfolio

The portfolio consists of bonds and stocks, which are perceived as imperfect substitutes. Bonds of different origin, yielding the same net interest rate (ρ_b), are considered perfect substitutes.

The same holds for domestic and foreign equities. Total wealth a is specified as a CES-composite of aggregate bonds b and equities e :

$$a = \left[\alpha_s \frac{-1}{\sigma_s} b^{\frac{\sigma_s+1}{\sigma_s}} + (1-\alpha_s) \frac{-1}{\sigma_s} e^{\frac{\sigma_s+1}{\sigma_s}} \right]^{\frac{\sigma_s}{\sigma_s+1}} \quad (1.15)$$

where α_s is a taste parameter and σ_s the substitution elasticity between bonds and stocks. The total (after tax) return on the portfolio satisfies:

$$\rho_s a = \rho_b b + \rho_e e \quad (1.16)$$

where ρ_x denotes the gross after tax rate of return on asset composite x ($x = \{s, b, e\}$). The optimal portfolio composition is found by maximizing (1.16) subject to (1.15), where the Lagrange multiplier is seen to equal the total rate of return ρ_s . The first-order conditions imply:

$$b = \left(\frac{\rho_b}{\rho_s} \right)^{\sigma_s} \alpha_s a \quad (1.17)$$

$$e = \left(\frac{\rho_e}{\rho_s} \right)^{\sigma_s} (1-\alpha_s) a \quad (1.18)$$

$$\rho_s = \left[\alpha_s \rho_b^{\sigma_s+1} + (1-\alpha_s) \rho_e^{\sigma_s+1} \right]^{\frac{1}{\sigma_s+1}} \quad (1.19)$$

⁴ In the gams-program we fix $A_l(0) = 1$ in the base year.

In the general case holds that $b + e \leq a$, meaning that a fraction of wealth is lost in making the aggregate.

Proof. From (1.17)-(1.19), one can derive that

$$\frac{b + e}{a} = \frac{\alpha_s \hat{\rho}^{\sigma_s} + (1 - \alpha_s)}{\Gamma}$$

with $\hat{\rho} \equiv \rho_b / \rho_e$

$$\Gamma \equiv [\alpha_s \hat{\rho}^{\sigma_s + 1} + (1 - \alpha_s)]^{\frac{\sigma_s}{\sigma_s + 1}}$$

After some manipulations, one gets

$$\begin{aligned} \frac{\partial (b + e) / a}{\partial \hat{\rho}} &= \frac{\alpha_s \sigma_s \hat{\rho}^{\sigma_s - 1}}{\Gamma} - \frac{\alpha_s \hat{\rho}^{\sigma_s} + (1 - \alpha_s)}{\Gamma^2} \sigma_s \Gamma^{-1 / \sigma_s} \alpha_s \hat{\rho}^{\sigma_s} \\ &= \frac{\alpha_s \sigma_s \hat{\rho}^{\sigma_s - 1} \Gamma^{-1 / \sigma_s}}{\Gamma^2} (1 - \alpha_s) (1 - \hat{\rho}) \end{aligned}$$

implying that

$$\begin{array}{ccc} < 1 & > 0 & < 1 \\ \hat{\rho} = 1 & \Rightarrow \frac{\partial (b + e) / a}{\partial \hat{\rho}} = 0 & \Rightarrow \frac{b + e}{a} = 1 \\ > 1 & < 0 & < 1 \end{array}$$

■

1.1.4 Taxation of portfolio income

Capital income is assumed to be only taxed in the country of residence. Tax authorities have full information about these income flows. Dividends, capital gains and interest income from bonds are taxed at the rate τ_d , τ_g and τ_b , respectively.

The after-tax rate of return on bonds is then by definition equal to:

$$\rho_b = 1 + r_b = 1 + \hat{r}_{wb} (1 - \tau_b) \quad (1.20)$$

where \hat{r}_{wb} is the world rate of return on bonds. The net return on equity $r_e = \rho_e - 1$ will be derived below in equation (1.56).

1.1.5 Aggregate consumption, wealth and savings (in a given period)

Aggregate consumption (per capita young) grows at rate g_a if and only if

$c^y(t, t) = (1 + g_a)c^y(t - 1, t - 1)$. We have already assumed that the consumption profile for each young person is $c^y(t - 1, t) = (1 + g_a)c^y(t - 1, t - 1)$. This implies that in a given period all young persons (of every birth year) consume the same amount $c^y(t)$ and every old person consumes $c^o(t)$. In per capita terms, total consumption equals:⁵

$$c(t) = \sum_s \omega^y(t - s) \left[c^y(t - s, t) + \frac{c^o(t - T - s, t)}{(1 + g_n)^T} \right]$$

⁵ The assumptions on population in section 1.1.1 imply:

$$N^i(-s) = \frac{(1 + g_n)^{-s}}{\sum_s (1 + g_n)^{-s}} N^i, \quad i = o, y$$

$$= c^y(t) + \frac{c^o(t)}{(1+g_n)^T} \quad (1.21)$$

Aggregate wealth is less straightforward, as it is not uniform across generations. Observe from equation (1.6) that for each household of the young generation holds:

$$\begin{aligned} a(t-i, t) &= s_n^y(t-i, t-i) \left[\frac{\rho_s^{i+1} - (1+g_a)^{i+1}}{\rho_s - (1+g_a)} \right] \\ &= s_n^y(t, t) (1+g_a)^{-i} \left[\frac{\rho_s^{i+1} - (1+g_a)^{i+1}}{\rho_s - (1+g_a)} \right] \\ &= s_n^y(t, t) \left[\frac{\theta^{i+1} - 1}{\theta - 1} \right], \quad i = 0, \dots, T-1 \end{aligned}$$

where we define $\theta \equiv \rho_s / (1+g_a)$. Similarly, for the old generation, using

$a(t-i, t) = (1+g_a)^{-i} a(t, t+i)$ in equation (1.7) implies:

$$\begin{aligned} a(t-T-i, t) &= (1+g_a)^{-(T+i)} a(t, t+T+i) \\ &= (1+g_a)^{-(T+i)} \left[\rho_s^{i+1} a(t, t+T-1) + \frac{\rho_s^{i+1} - (1+g_a)^{i+1}}{\rho_s - (1+g_a)} s_n^o(t, t+T) \right] \\ &= \left[\theta^{i+1} \frac{\theta^T - 1}{\theta - 1} - \theta^T \frac{\theta^{i+1} - 1}{\theta - 1} \right] s_n^y(t, t) \end{aligned}$$

Total wealth AS is the summation over all young and old cohorts:

$$\begin{aligned} AS(t) &= N^y \sum_{i=0}^{T-1} \left[\omega^y(-i) a(t-i, t) + \omega^y(-i) \frac{a(t-T-i, t)}{(1+g_n)^T} \right] \\ &= N^y s_n^y(t, t) \chi(g_a, g_n, \rho_s) \\ \chi(g_a, g_n, \rho_s) &\equiv \sum_{i=0}^{T-1} \omega^y(-i) \left[\frac{\theta^{i+1} - 1}{\theta - 1} + \frac{1}{(1+g_n)^T} \left(\theta^{i+1} \frac{\theta^T - 1}{\theta - 1} - \theta^T \frac{\theta^{i+1} - 1}{\theta - 1} \right) \right] \end{aligned} \quad (1.22)$$

such that in per capita terms:

$$as(t) = [\bar{w}(t)l + tr^y(t) - c^y(t)] \chi(g_a, g_n, \rho_s) \quad (1.23)$$

where χ is a (country-specific) parameter, depending on population growth, productivity growth and the return on savings. This parameter does not depend on time, unlike N^y and s^y . This implies that aggregate savings including interest income are:

$$S(t) = AS(t) - AS(t-1) = AS(t) \left[1 - \frac{1}{(1+g_n)(1+g_a)} \right] \quad (1.24)$$

which equals zero if productivity and population growth are both zero.

$$\begin{aligned} \omega^y(-s) &\equiv \frac{(1+g_n)^{-s}}{\sum_s (1+g_n)^{-s}} \quad \sum_s \omega^y(-s) = 1 \\ \omega^y(-s) &= \frac{N^y(-s)}{N^y} = \frac{N^y(-T-s)}{N^o} \end{aligned}$$

where s is the generation born s or $T+s$ periods ago, $N^i(-s)$ is the size of the s -year old age-cohort and $\omega^y(-s)$ is the relative size of this cohort (as fraction of the young population).

Saving rate As an additional piece of evidence, we might use the saving rate to calibrate the model. One common definition of the saving rate is savings as fraction of households disposable income. Savings are defined in equation (1.24). Disposable income is the sum over income of young and old generations. Young households earn wage income, receive transfers and build up assets for which they get interest income. Old households receive profit-income, transfers and interest income. Note that only the interest income varies between households. This implies that:

$$\begin{aligned}
Y_d(t) &= N^y \sum_{i=0}^{T-1} \omega^y(-i) [\bar{w}(t)l + tr^y(t) + (\rho_s - 1)a(t - i, t - 1)] \\
&\quad + N^o \sum_{i=0}^{T-1} \omega^o(-i) [\pi^o(t) + tr^o(t) + (\rho_s - 1)a(t - T - i, t - 1)] \\
&= N^y \left[\bar{w}(t)l + tr^y(t) + \frac{\pi^o(t) + tr^o(t)}{(1 + g_n)^T} \right] + (\rho_s - 1)AS(t - 1)
\end{aligned} \tag{1.25}$$

Combined with equations (1.23) and (1.24) we obtain the saving rate:

$$\omega_{sy} = \frac{S(t)}{Y_d(t)} = \frac{\left[1 - \frac{1}{(1+g_n)(1+g_a)} \right] \chi(g_a, g_n, \rho_s) [\bar{w}l + tr^y - (1 - \tau_c)c^y]}{\bar{w}l + tr^y + \frac{\pi^o + tr^o}{(1+g_n)^T} + (\rho_s - 1)\chi(g_a, g_n, \rho_s) [\bar{w}l + tr^y - (1 - \tau_c)c^y]} \tag{1.26}$$

Compensating variation In simulations, we compare the welfare impact of tax changes by calculating the compensating variation (cv). The cv is calculated as the change in transfers to young households required to compensate the change in welfare. The system of equations used to calculate cv consists of the definition of welfare in (1.2) and (1.3), the optimal response of labour and consumption in (1.13) and (1.14), and the budget equation (1.9).

The interpretation of the cv is hampered by the fact that a welfare gain is represented by a negative compensation. In the output tables we overcome this by reporting the $cv_{gain} \equiv -100 \frac{cv}{y}$, where y is GDP in the base case.

1.2 Firms

Three types of firms are active in each country: pure domestic firms, headquarters of multinationals and subsidiaries of foreign multinationals. Firm's types are represented by the superscripts d , m and f , respectively. Each country is endowed with a stock of a fixed factor, named 'location specific capital'. Its size is assumed proportional to the generation size N^y to avoid that productivity differentials would arise from differences in country size (cfr. Sørensen (2001a), p. 7). To be precise, this factor is called fixed since its supply is perfectly inelastic. An individual firm can choose the amount of this factor optimally. In equilibrium, the fixed factor is paid its marginal productivity. The three firm types are successively discussed in the following paragraphs.

1.2.1 Domestic firms

The marginal investor maximises the present value of the representative firm, which is equal to the discounted stream of expected dividends. Uncertainty applies to the productivity of firms in each year, as will be explained below.⁶ The discount rate of investors residing in different countries differs due to varying tax rates on capital income. It implies that the present value differs between investors. To single out a unique investor, we assume that the marginal investor is domestic.

The gross return on equities in period t consists of dividends and capital gains:

$$\hat{r}_{we} V_t^d = Div_t^d + V_{t+1}^d - V_t^d \quad (1.27)$$

where \hat{r}_{we} is the world rate of return on equity, V^d is the value of the firm and Div^d the distributed profits. The net return on equity $r_e(i, j) = \rho_e(i, j) - 1$ follows from subtracting personal taxes:

$$\begin{aligned} r_e(i, j) V_t^d(i, j) &= \hat{r}_{we}(j) V_t^d(i, j) - \tau_d(i) Div_t^d(i, j) - \tau_g(i) (V_{t+1}^d(i, j) - V_t^d(i, j)) \implies \\ r_e(i, j) V_t^d(j) &= (1 - \tau_d(i)) Div_t^d(j) + (1 - \tau_g(i)) (V_{t+1}^d(j) - V_t^d(j)) \end{aligned} \quad (1.28)$$

The second line follows from the assumption that each investor irrespective of its residence country receives the same dividend and capital gain per share, with $Div^d(j) = \sum_i Div^d(i, j)$ and $V^d(j) = \sum_i V^d(i, j)$. This equation shows that investors who face different tax rates will value firms differently. In principle, investors who require the lowest net return are willing to pay the most for an equity. Under the assumption that the marginal investor is domestic ($i = j$), recursive substitution of (1.28) shows that the value of the firm equals the sum over the present value of the dividends:

$$\begin{aligned} V_t^d(j) &= \sum_{s=t}^{\infty} \Lambda(j) Div_s^d(j) R_s(j) \quad \text{with } R_s(j) \equiv \frac{1}{(1 + \bar{r}_e(j))^{s-t+1}} \\ \bar{r}_e(j) &\equiv \frac{r_e(j, j)}{1 - \tau_g(j)} \\ \Lambda(j) &\equiv \frac{1 - \tau_d(j)}{1 - \tau_g(j)} \end{aligned} \quad (1.29)$$

where \bar{r}_e represents the discount rate relevant for firm's decisions. From here onward, we drop the country index, since both the firm and the marginal investor reside in country j .

Production function Maximization of the firm's value requires that an expression for the dividends is derived.⁷ The first key ingredient is the production function, in which we introduce uncertainty. With probability q a firm benefits from a good event ($x = q$) with high productivity,

⁶ We will only add expectatoinis explicitly when essential.

⁷ The following analysis can be found in e.g. Salinger and Summers (1983).

but with probability $1 - q$ it faces a bad event ($x = b$). For the representative domestic firm production is specified as:

$$Y^{d,x} = A^{d,x} \left(VA^{d,x} \right)^{\alpha_v^d} \quad \text{with } 0 < \alpha_v < 1 \quad (1.30)$$

$$A^{d,x} = \left(A_{0,x} \omega^d N^y \right)^{1 - \alpha_v^d}$$

where $Y^{d,x}$ denotes total output, such that $E(Y^d) = qY^{d,g} + (1 - q)Y^{d,b}$. $A^{d,x}$ is the output contribution of the fixed factor, VA^d value-added and α_v^d the share of value-added in production. The exogenous fraction of the fixed factor that is in use by domestic corporations is denoted by ω^d . Value-added is a CES-function of employment L^d and capital K^d :

$$VA^{d,x} = A_{0,x} \left[\alpha_{vl}^d \left(L^d \right)^{\frac{\sigma_v^d - 1}{\sigma_v^d}} + \alpha_{vk}^d \left(K^d \right)^{\frac{\sigma_v^d - 1}{\sigma_v^d}} \right]^{\frac{\sigma_v^d}{\sigma_v^d - 1}} \quad (1.31)$$

where α_{vl}^d is a share parameter and σ_v^d is the substitution elasticity between labour and capital. Note that we do not distinguish employment and capital between the good and bad case, as we assume that firms has to decide on their inputs before the productivity shock occurs. The total factor productivity (TFP) level A_0 serves two purposes: it facilitates the calibration of GDP and it allows for the introduction of productivity growth. We assume that TFP is uniform within a country across the three firm-types. In addition, we assume that its growth rate g_a is uniform across countries. We impose steady growth with $g_k = g_y$ and employment growth equal to population growth g_n . Equations (1.30) and (1.31) implies $g_y = g_{va} = (1 + g_a)(1 + g_n) - 1$.⁸ Marginal productivities are derived as:

$$\frac{\partial Y^{d,x}}{\partial L^d} = \left(\alpha_v^d \frac{Y^{d,x}}{VA^{d,x}} \right) \alpha_{vl}^d A_{0,x}^{1 - 1/\sigma_v^d} \left(\frac{VA^{d,x}}{L^d} \right)^{1/\sigma_v^d} \quad (1.32)$$

$$\frac{\partial Y^{d,x}}{\partial K^d} = \left(\alpha_v^d \frac{Y^{d,x}}{VA^{d,x}} \right) \alpha_{vk}^d A_{0,x}^{1 - 1/\sigma_v^d} \left(\frac{VA^{d,x}}{K^d} \right)^{1/\sigma_v^d} \quad (1.33)$$

Debt or equity financing The second ingredient for the expression of dividends is the determination of the debt ratio. Investment can be financed by issuing bonds or by retaining profits (issuing new shares is not considered).⁹ The world gross real rates of return on bonds and equities are denoted by \hat{r}_{wb} and \hat{r}_{we} , respectively. First, an interior solution for the financing mix is obtained by assuming that both debt and equity financing are extremely costly at the corner:

$$c_b^i (d_b^i) = \chi_0 (1 - d_b^i)^{-(1 - \varepsilon_b)} (d_b^i)^{-\varepsilon_b} - c_{b,0}^i \quad i = d, m, f \quad \text{with } \chi_0, \varepsilon_b > 0 \quad (1.34)$$

⁸ The growth rate g_y applies on the steady growth path to $Y, wL, D, I, \Pi, \hat{\Pi}, Div$.

⁹ In the model of Auerbach and King (1983) individual firms will either choose debt or equity financing, but an interior solution with both debt and equity financing at the firm-level requires very strong restrictions. At the industry or macro-level, an interior solution is feasible if firms are heterogenous, with varying preference for debt or equity financing (like with varying risk aversion).

where d_b^i is the firm's debt-asset ratio and c_b^i is the cost of financial distress per unit of capital. This cost represents the output which is lost as financial decisions distract managers from productive activities. The partial derivative of this cost w.r.t. the debt-ratio is:

$$\begin{aligned}\frac{\partial c_b^i}{\partial d_b^i} &= \left[(1 - \varepsilon_b) (1 - d_b^i)^{-1} - \varepsilon_b (d_b^i)^{-1} \right] \chi_0 (1 - d_b^i)^{-(1-\varepsilon_b)} (d_b^i)^{-\varepsilon_b} \\ &= \left[(1 - \varepsilon_b) (1 - d_b^i)^{-1} - \varepsilon_b (d_b^i)^{-1} \right] (c_b^i + c_{b,0}^i)\end{aligned}\quad (1.35)$$

This implies that the cost function has its minimum at:

$$d_{b,min}^i = \varepsilon_b \quad \text{with } c_b^i(d_{b,min}^i) = \chi_0 (1 - \varepsilon_b)^{-(1-\varepsilon_b)} (\varepsilon_b)^{-\varepsilon_b} - c_{b,0}^i$$

When $c_{b,0}^i$ is used to set this minimum equal to zero, we can calibrate the actual debt-asset ratio with the parameters ε_b and χ_0 . The sensitivity of the cost function w.r.t. changes in the debt-ratio will depend on both parameters, but mainly on χ_0 , whereas ε_b first of all represents the debt rate at which financial distress costs are minimized.

Dividends Firms make profits after a good event, but losses after a bad event. The losses made in period $t - 1$ can be carried forward to period t . The notional (i.e. before loss carry forward) tax base in both cases $\widehat{\Pi}^{d,x}$ of corporate taxation is defined as:

$$\widehat{\Pi}^{d,x} = Y^{d,x} - wL^d - \left(\beta_b d_b^d \widehat{R}_{wb} + c_b^d \right) K^d - \left(\delta_t + \beta_e (1 - d_b^d) \bar{R}_e \right) D^d - \varphi I^d \quad (1.36)$$

where β_b is the deductible fraction of interest payments, β_e is the deductible fraction of equity payments, δ_t the depreciation rate of capital for tax purposes, D^d the stock of depreciation allowances and φI is the fraction of investments which can be expensed immediately.¹⁰ Nominal returns to debt and equity are (or might be) deductible: $\widehat{R}_{wb} \equiv (1 + \widehat{r}_{wb})(1 + \pi) - 1$ and $\bar{R}_e \equiv (1 + \bar{r}_e)(1 + \pi) - 1$. Only positive profits will be taxed, where losses of the previous period (which have occurred with probability $1 - q$ are carried forward (F):

$$F_t^d = - \frac{\widehat{\Pi}_{t-1}^{d,b}}{(1 + \pi)(1 + g_y)} \quad (1.37)$$

$$\widehat{\Pi}_t^d = \widehat{\Pi}_t^{d,g} - (1 - q)F_t^d \quad (1.38)$$

When exponential depreciation is allowed for tax purposes, the accumulation of depreciation rights is similarly specified as the accumulation of physical capital:

$$\text{fiscal : } D_{t+1}^d = I_t^d + (1 - \delta_t)D_t^d \quad (1.39)$$

$$\text{economic : } K_{t+1}^d = I_t^d + (1 - \delta_k)K_t^d \quad (1.40)$$

¹⁰ Notice that the tax base includes fixed-factor income, which justifies a positive corporate tax rate.

where I^d stands for investment and δ_k for the real depreciation rate¹¹. Corporate taxes are equal to $\tau_\pi^d \widehat{\Pi}^d$.¹² Dividends now follow from the cash flow restriction:

$$E(Div_t^d) = E(Y_t^d) - w_t L_t^d - \left(d_{b,t}^d \hat{r}_{wb} + c_{b,t}^d \right) K_t^d - \Pi_t^d - q \tau_\pi^d \widehat{\Pi}_t^d - I_t^d + d_{b,t+1}^d K_{t+1}^d - d_{b,t}^d K_t^d \quad (1.41)$$

where Π^d denote returns to fixed factors.

Profit maximization The firm is assumed to maximize its value (1.29), subject to the accumulation equations (1.39)-(1.40). The Lagrange function is written as:

$$L = \sum_{s=t}^{\infty} \left\{ \Lambda E(Div_s^d) - \lambda_{s+1}^d \left(D_{s+1}^d - I_s^d - (1 - \delta_i) D_s^d \right) - \mu_{s+1}^d \left(K_{s+1}^d - I_s^d - (1 - \delta_k) K_s^d \right) \right\} R_s \quad (1.42)$$

The first order condition of L^d gives the marginal productivity condition:

$$\frac{\partial Y^d}{\partial L^d} = \frac{(1 - \theta) \tau_\pi}{(1 - \tau_\pi) A^F} w = w \text{ if } q = 1 \text{ or } A_{0,b} = A_{0,g} \quad (1.43)$$

where

$$\theta \equiv q \left(1 + \frac{1 - q}{1 + \bar{r}_e} \right) \quad A^F \equiv q + (1 - q) \frac{A^{d,b}}{A^{d,g}} + \frac{\tau_\pi}{1 - \tau_\pi} (1 - \theta) \frac{A^{d,b}}{A^{d,g}} \quad (1.44)$$

With the CRS production function and perfect competition, each production factor is paid its after tax marginal return. This will also hold for the fixed production factor ($\omega^d N^y$):

$$\Pi^d = \left(1 - \tau_\pi^d \right) \frac{\partial Y^d}{\partial (\omega^d N^y)} \omega^d N^y = A^F \left(1 - \tau_\pi^d \right) \left(1 - \alpha_v^d \right) Y^d \quad (1.45)$$

The optimal debt ratio has to satisfy the condition:

$$\frac{\partial c_{b,t}^d}{\partial d_{b,t}^d} = \frac{\bar{r}_e - \tau_\pi^d \beta_e \bar{R}_e D / K}{1 - \tau_\pi^d} - \left(\frac{\hat{r}_{wb} - \tau_\pi^d \beta_b \hat{R}_{wb}}{1 - \tau_\pi^d} \right) \quad (1.46)$$

Since debt normally carries the lowest financing cost ($\bar{r}_e > \hat{r}_{wb}$), condition (1.46) generally implies that $d_b^d > \varepsilon_b$ and $\partial c_b^d / \partial d_b^d > 0$.¹³ The first order condition of investment gives

$$(1 - \varphi) \lambda^d + \mu^d = (1 - \theta \tau_\pi \varphi) \Lambda \quad (1.47)$$

¹¹ The specification (1.39) yields a similar optimal condition for capital as in Sørensen (2001b). One could favour a change of the time index for investment into $t + 1$.

¹² A difference with Sørensen (2001b) can be noted. The value of the depreciation allowances in OECDTAX (see (67), in our notation) is

$$\delta_i D = \frac{\delta_i (\delta_k + r)}{(\delta_i + r)} K$$

Since D and K grow at rate g_y in the steady state, (1.39)-(1.40) imply

$$\delta_i D = \frac{\delta_i (\delta_k + g_y)}{(\delta_i + g_y)} K$$

Normally holds that $r > g_y$ and $\delta_i > \delta_k$, implying that the tax allowance in OECDTAX is larger (for a given K).

¹³ Instead of \bar{r}_e , Sørensen (2001b) uses \hat{r}_e in the equivalent of (1.46). Normally, $\bar{r}_e < \hat{r}_e$ holds.

The condition for the state variable D^d is:

$$\left[\Lambda \tau_\pi^d (\delta_t + \beta_e (1 - d_b^e) \bar{R}_e) + (1 - \delta_t) \lambda_{s+1}^d \right] R_s = \lambda_s^d R_{s-1} (1 + \pi) \quad (1.48)$$

At the steady growth path the shadow value λ^d is constant at the value:

$$\frac{\lambda^d}{\Lambda} = \frac{\theta \tau_\pi^d (\delta_t + \beta_e (1 - d_b^e) \bar{R}_e)}{\bar{R}_e + \delta_t} \quad (1.49)$$

which is the present value of the stream of depreciation allowances for one unit of capital.

Finally, the first order condition for capital can be derived as:

$$\begin{aligned} \left[\Lambda A^F \frac{\partial Y_t^d}{\partial K_t^d} (1 - \tau_\pi^d) - \Lambda c_{b,t}^d - \Lambda d_{b,t}^d (\hat{r}_{wb} - \theta \tau_\pi^d \beta_b \hat{R}_{wb}) - \Lambda d_{b,t}^d + \mu_{t+1}^d (1 - \delta_k) \right] R_t \\ = - \left(\Lambda d_{b,t}^d - \mu_t^d \right) R_{t-1} \end{aligned} \quad (1.50)$$

Use (1.47) to simplify this expression to:

$$\frac{\partial Y^d}{\partial K^d} = c^d \quad (1.51)$$

where we define the user cost of capital stock c^d and the marginal cost of finance r^d as:¹⁴

$$\begin{aligned} A^F (1 - \tau_\pi) c^d \equiv r^d + \delta_k + (1 - d_b) \theta \tau_\pi \beta_e \bar{R}_e \\ - \theta \tau_\pi [\varphi (\bar{R}_e + \delta_t) + (1 - \varphi) (\delta_t + \beta_e (1 - d_b) \bar{R}_e)] \left(\frac{\bar{r}_e + \delta_k}{\bar{R}_e + \delta_t} \right) \end{aligned} \quad (1.52)$$

$$r^d \equiv d_b^d (\hat{r}_{wb} - \tau_\pi^d \beta_b \hat{R}_{wb}) + (1 - d_b^d) (\bar{r}_e - \tau_\pi^d \beta_e \hat{R}_e) + c_b^d \quad (1.53)$$

The value of the firm is shown to be equal to the sum of the values of the physical and the accounting stock of capital, see Salinger and Summers (1983, eq. (14)):

$$V^d = \Lambda (1 - d_b^d) K^d + \lambda^d (D^d - K^d) \quad (1.54)$$

As dividends grow at rate $g_y = (1 + g_a)(1 + g_n) - 1$ at the *steady growth path*, an alternative expression is easily found:

$$V^d = \frac{\Lambda Div^d}{\bar{r}_e - g_y} \quad \text{with } \bar{r}_e > g_y \quad (1.55)$$

Furthermore, $\Delta V^d = g_y V^d$ in the steady state implies $Div^d / V^d = \hat{r}_{we} - g_y$ in view of (1.27).

Substitution in (1.28) yields:

$$r_e(i, j) = (1 - \tau_d(i)) (\hat{r}_{we}(j) - g_y) + (1 - \tau_g(i)) g_y \quad (1.56)$$

$$\bar{r}_e(j) = \Lambda(j) \hat{r}_{we}(j) + (1 - \Lambda(j)) g_y \quad (1.57)$$

¹⁴ Notice that the only difference with the corresponding condition (69) in Sørensen (2001b) concerns the effect of depreciation allowances. Whereas the discount rate r^d is assumed for depreciation allowances in OECDTAX, the rate \bar{r}_e applies in our dynamic context.

1.2.2 Multinational parent company

The domestic operations of the multinationals are analogously specified:

$$Y^{m,x} = A^{m,x} (VA^{m,x})^{\alpha_v^m} \quad \text{with } 0 < \alpha_v^m < 1 \quad (1.58)$$

where $Y^{m,x}$ denotes total output, $A^{m,x}$ the output contribution of the fixed factor, and $VA^{m,x}$ value-added. Multinationals hold fraction $\omega^m = 1 - \omega^d$ of the fixed factor. Value-added is a CES-function of employment L^m and capital K^m :

$$A^{m,x} = (A_{0,x} \omega^m N^y)^{1-\alpha_v^m} \quad (1.59)$$

$$VA^{m,x} = A_{0,x} \left[\alpha_{vl}^m (L^m)^{\frac{\sigma_v^m-1}{\sigma_v^m}} + \alpha_{vk}^m (K^m)^{\frac{\sigma_v^m-1}{\sigma_v^m}} \right]^{\frac{\sigma_v^m}{\sigma_v^m-1}} \quad (1.60)$$

Marginal productivities are similar to (1.32) and (1.33). When the corporation's debt-asset ratio d_b^m deviates from ε_b , it has to pay financial distress costs, cf. (1.34). The marginal cost of finance is defined as:

$$r^m \equiv d_{b,t} (\hat{r}_{wb} - \beta_b \theta \tau_\pi \hat{R}_{wb}) + (1 - d_b) (\bar{r}_e - \beta_e \theta \tau_\pi \bar{R}_e) + c_b \quad (1.61)$$

The parent company supplies $Q(j)$ units as an input to its foreign subsidiary j . When the tax rate on profits differs between both countries, transfer pricing might be attractive to shift taxable profits between the jurisdictions (Sørensen (2001b), p. 24). However, charging a different price than the real cost (i.e. $p_q \neq 1$) involves a type of organizational costs. The cost arising from a distorted transfer price is assumed to be:

$$c_q = \frac{|p_q - 1|^{1+\varepsilon_q}}{1 + \varepsilon_q} \quad \text{with } \varepsilon_q > 0 \quad (1.62)$$

$$\Rightarrow \frac{\partial c_q}{\partial p_q} = \text{sign}(p_q - 1) |p_q - 1|^{\varepsilon_q}$$

The notional corporate tax base is given by

$$\begin{aligned} \hat{\Pi}^{m,x} &= Y^{m,x} - wL^m + \sum_{j \neq i} (p_q(j) - 1 - c_q(j)) Q(j) - (\beta_b d_b^m \hat{r}_{wb} + c_b^m) K^m \\ &\quad - (\delta_i + \beta_e (1 - d_b^m) \bar{R}_e) D^m - \varphi I^m \end{aligned} \quad (1.63)$$

Again, only positive profits will be taxed, where losses of the previous period (which have occurred with probability $1 - q$ are carried forward (F), but losses of subsidiaries will not be consolidated:

$$F_t^m = - \frac{\hat{\Pi}_{t-1}^{m,b}}{(1 + \pi)(1 + g_y)} \quad (1.64)$$

$$\hat{\Pi}_t^m = \hat{\Pi}_t^{m,s} - (1 - q) F_t^m \quad (1.65)$$

The dividends originating from domestic operations are:

$$\begin{aligned} E(\text{Div}_t^{mm}) &= E(Y_t^m) - w_t L_t^m + \sum_{j \neq i} (p_q(j) - 1 - c_q(j)) Q(j) - (d_{b,t}^m \hat{r}_{wb} + c_{b,t}^m) K_t^d - \Pi_t^m \\ &\quad - q \tau_\pi \hat{\Pi}_t^m - I_t^m + d_{b,t+1}^m K_{t+1}^m - d_{b,t}^m K_t^m \end{aligned} \quad (1.66)$$

The optimal decisions of multinationals follow from the maximization of its *total* value, which is described in the following paragraph.

1.2.3 Multinational subsidiaries

Production of the subsidiary in country j is given by:

$$Y^{f,x}(j) = A^{f,x}(j) A_0^{\alpha_q} Q(j)^{\alpha_q} VA^{f,x}(j)^{\alpha_v^f} \quad \text{with } 0 < \alpha_q + \alpha_v^f < 1 \quad (1.67)$$

$$A^{f,x} = \left(A_{0,x} \omega^f N^y \right)^{1 - \alpha_v^f - \alpha_q} \quad (1.68)$$

$$VA^{f,x}(j) = A_{0,x} \left[\alpha_{vl}^f \left(L^f(j) \right)^{\frac{\sigma_v^f - 1}{\sigma_v^f}} + \alpha_{vk}^f \left(K^f(j) \right)^{\frac{\sigma_v^f - 1}{\sigma_v^f}} \right]^{\frac{\sigma_v^f}{\sigma_v^f - 1}} \quad (1.69)$$

where Y^f denotes total output, A^f the output contribution of the fixed factor, Q the intermediate input and VA^f value-added.

The equity of the subsidiary is assumed to be completely provided by its parent, implying that the equity cost equals the opportunity cost in the parent's country ($\bar{r}_e(i)$). The multinational finances the remaining fraction of the capital stock by issuing bonds at the cost \hat{r}_{wb} . The subsidiary's marginal cost of finance is written as

$$r^f(j) \equiv d_b^f(j) (1 - \tau_\pi(j)) \hat{r}_{wb} + \left(1 - d_b^f(j) \right) (\bar{r}_e(i) - \beta_e \theta \tau_\pi \bar{R}_e) + c_b^f(j) \quad (1.70)$$

where financial distress costs c_b^f are defined in equation (1.34). Its notional tax base is defined according to the foreign jurisdiction:

$$\begin{aligned} \hat{\Pi}^{f,x}(j) &= Y^{f,x}(j) - w(j)L^f(j) - p_q(j)Q(j) - \left(\beta_b(j)d_b^f(j)\hat{r}_{wb} + c_b^f(j) \right) K^f(j) \\ &\quad - \left(\delta_t + \beta_e(1 - d_b^f)\bar{R}_e \right) D^f(j) - \varphi(j)I^m(j) \end{aligned} \quad (1.71)$$

such that

$$F_t^f = - \frac{\hat{\Pi}_{t-1}^{f,b}}{(1 + \pi)(1 + g_y)} \quad (1.72)$$

$$\hat{\Pi}_t^f = \hat{\Pi}_t^{f:s} - (1 - q)F_t^f \quad (1.73)$$

Remaining profits flowing to the parent company follow as¹⁵

$$\begin{aligned} E(Div_t^{mf}(j)) &= E(Y_t^f(j)) - w_t(j)L_t^f(j) - p_q(j)Q(j) - \left(d_{b,t}^f(j)\hat{r}_{wb} + c_{b,t}^f(j) \right) K_t^f \\ &\quad - q\Pi_t^f(j) - \tau_\pi^f(j)\hat{\Pi}_t^f(j) - I_t^f(j) + d_{b,t+1}^f(j)K_{t+1}^f(j) - d_{b,t}^f(j)K_t^f(j) \end{aligned} \quad (1.74)$$

Profit maximization The multinational maximizes the value

$$V_t^m = \sum_{s=t}^{\infty} \Lambda Div_s^m R_s = \sum_{s=t}^{\infty} \Lambda \left[Div_s^{mm} + \sum_{j \neq i} Div_s^{mf}(j) \right] R_s \quad (1.75)$$

¹⁵ Pure profits of foreign subsidiaries are assumed to accrue to the old generation living in the parent country.

The optimal factor demands and debt ratio are derived similarly as for the domestic firms. For labor (cf. 1.43):

$$\frac{\partial Y^m}{\partial L^m} = \frac{(1-\theta)\tau_\pi}{(1-\tau_\pi)A^F} w \quad (1.76)$$

$$\frac{\partial Y^f(j)}{\partial L^f(j)} = \frac{(1-\theta)\tau_\pi}{(1-\tau_\pi)A^F} w(j) \quad (1.77)$$

For investment:

$$\frac{\lambda^m}{\Lambda} = \frac{\theta\tau_\pi(\delta_t + \beta_e(1-d_b^e)\bar{R}_e)}{\delta_t + \bar{R}_e} \quad (1.78)$$

$$\frac{\lambda^f(j)}{\Lambda} = \frac{\theta\tau_\pi(j)(\delta_t(j) + \beta_e(j)(1-d_b^e)\bar{R}_e)}{\delta_t(j) + \bar{R}_e} \quad (1.79)$$

For capital:

$$A^F(1-\tau_\pi)c^m \equiv r^m + \delta_k + (1-d_b)\theta\tau_\pi\beta_e\bar{R}_e \quad (1.80)$$

$$\begin{aligned} & - \theta\tau_\pi[\delta_t + \varphi\bar{R}_e + (1-\varphi)\beta_e(1-d_b)\bar{R}_e] \frac{\bar{r}_e + \delta_k}{\bar{R}_e + \delta_t} \\ A^F(j)(1-\tau_\pi(j))c^f(j) & \equiv r^f(j) + \delta_k + (1-d_b)\theta\tau_\pi(j)\beta_e(j)\bar{R}_e \quad (1.81) \\ & - \theta\tau_\pi(j)[\delta_t(j) + \varphi(j)\bar{R}_e + (1-\varphi(j))\beta_e(j)(1-d_b)\bar{R}_e] \frac{\bar{r}_e + \delta_k}{\bar{R}_e + \delta_t(j)} \end{aligned}$$

For the fixed factor:

$$\Pi^m = (1-\alpha_v^m)(1-\tau_\pi^m)A^F Y^m \quad (1.82)$$

$$\Pi^f(j) = (1-\alpha_q - \alpha_v^f)(1-\tau_\pi^f(j))A^F Y^f(j) \quad (1.83)$$

For the debt ratio:

$$\frac{\partial c_{b,t}^i}{\partial d_{b,t}^i} = \frac{\bar{r}_e - \tau_\pi^i \beta_e \bar{R}_e D/K}{1 - \tau_\pi^i} - \left(\frac{\hat{r}_{wb} - \tau_\pi^i \beta_b \hat{R}_{wb}}{1 - \tau_\pi^i} \right), \quad i = m, f \quad (1.84)$$

In addition, the expressions for intermediate inputs and corresponding transfer prices are derived as

$$A^F \frac{\partial Y^f(j)}{\partial Q(j)} (1 - \tau_\pi^f(j)) = \theta p_q(j) (\tau_\pi^m - \tau_\pi^f(j)) + (1 + c_q(j))(1 - \theta\tau_\pi^m) \quad j \neq i \quad (1.85)$$

$$\frac{\partial c_q(j)}{\partial p_q(j)} (1 - \tau_\pi^m) = \tau_\pi^f(j) - \tau_\pi^m \quad (1.86)$$

From the last condition follows that the multinational shifts profits to the jurisdiction with the lowest tax rate, since $p_q(j) > (<) 1$ if $\tau_\pi^f(j) > (<) \tau_\pi^m$.

The first order conditions also imply that the value of the multinational equals the value of the stocks it owns:

$$V^m = \Lambda(1-d_b^m - \theta\tau_\pi^m\varphi)K^m + \lambda^m[(1+\pi)D^m - (1-\varphi)K^m] + \Lambda(1-q)q\tau F \quad (1.87)$$

Foreign direct investment (FDI) is defined as the equity-financed part of foreign capital:

$$FDI(i, j) = (1 - d_b^f(i, j)) K^f(i, j) \quad (1.88)$$

1.2.4 Aggregate production

Gross domestic product is defined as the sum of production of all firms in a country corrected for the value of intermediate inputs in foreign subsidiaries:

$$Y(i) \equiv q \left[Y^{d,g}(i) + Y^{m,g}(i) + \sum_{j \neq i} Y^{f,g}(j,i) \right] + (1-q) \left[Y^{d,b}(i) + Y^{m,b}(i) + \sum_{j \neq i} Y^{f,b}(j,i) \right] - \sum_{j \neq i} p_q(j,i) Q(j,i) \quad (1.89)$$

1.2.5 Marginal effective tax rate

For calibration as well as for output purposes, the effective tax rate is calculated. An effective tax rate is defined as the relative difference between pre and post tax capital costs. The effective *marginal* tax rate (τ_e^x) is relevant for marginal investment decisions. In our model the effective marginal tax rate equals:

$$\tau_e^x = \frac{c^x - (c^x | \tau_\pi = 0)}{c^x}, \quad x = d, m, f \quad (1.90)$$

where c^x is defined in (1.52), (1.80) and (1.81).

1.2.6 Tax haven

This section introduces profit shifting to a tax haven. We assume that MNEs (not domestic firms) can shift part of their tax base to a tax haven. MNEs therefore know by their all their decisions, that only a fraction of their return is taxed against the statutory tax rate (at source).

We assume that a fraction θ of the tax base is shifted to the tax haven, where it is taxed at rate τ_π^h . This implies that the benefits from tax shifting are:

$$b^{th} = \theta (\tau_\pi^m - \tau_\pi^h) \hat{\Pi} \quad (1.91)$$

Tax shifting is, however, costly for firms. We specify the cost of profit shifting as:

$$c^{th}(\theta) = A^{-1/\gamma} \frac{\theta^{1+1/\gamma}}{1+1/\gamma} \hat{\Pi} \quad (1.92)$$

The optimal choice of the fraction of tax shifting is then:

$$\theta^* = A (\tau_\pi^m - \tau_\pi^h)^\gamma \quad (1.93)$$

This implies that the net reduction in tax-payments for firms become:

$$b^{th} - c^{th} = \tau_\pi^m A \left(1 - \frac{1}{1+\gamma} \right) (\tau_\pi^m - \tau_\pi^h)^{\gamma+1} \hat{\Pi} \quad (1.94)$$

The 'loss' in tax revenues for the domestic government is:

$$\tau_\pi^m \theta^* \hat{\Pi} = \tau_\pi^m A (\tau_\pi^m - \tau_\pi^h)^\gamma \hat{\Pi} \quad (1.95)$$

The gain for the tax haven is equal to:

$$\tau_\pi^h \theta^* \hat{\Pi} = \tau_\pi^h A (\tau_\pi^m - \tau_\pi^h)^\gamma \hat{\Pi} \quad (1.96)$$

For MNEs, profit shifting to the tax haven implies that the effective statutory tax rate, which determines the optimal demand for production factors reduces with a factor $\theta^* \leq 1$.

1.2.7 Location choice

Another extension refers to location choice. The literature on foreign direct investment emphasises that investment is not only responsive to the cost of capital, but that also inframarginal investment and location choices are important. One reason may be that firms earn firm-specific economic rents that are mobile across borders. Such rents can be due to patents, brand names, specific managerial talents or market power. Firms then locate their affiliates in countries where the average effective tax rates are relatively low.

We do not explicitly model the origins of firm-specific economic rents. Instead, we endogenise the value of economic rents earned by a multinational in each location by making it dependent of the corporate tax rate. In particular, suppose that the multinational owns a firm-specific fixed factor ω , which it can allocate between two countries, ω_i and ω_j . If the firm maximizes the sum of profits in the two locations ($\Pi_i + \Pi_j$) given in equation (1.82), the first order condition with respect to the allocation of the fixed factor in country i reads as

$$\frac{d\Pi}{d\omega_i} = (1 - \tau_i)(1 - \alpha) \frac{dY_i}{d\omega_i} + (1 - \tau_j)(1 - \alpha) \frac{dY_j}{d\omega_j} \frac{d\omega_j}{d\omega_i} = 0 \quad (1.97)$$

Given the production function (1.58), this yields a simple expression for the marginal value of allocating the fixed factor in each of the two locations. Using this production structure, we can write the optimal share of the fixed factor in the two locations as:

$$\frac{\omega_j}{\omega_i} = \frac{VA_j}{VA_i} \left[\frac{1 - \tau_j}{1 - \tau_i} \right]^{\frac{1}{\alpha}} \quad (1.98)$$

where VA denotes the composite input of labour and capital. Hence, the share of the fixed factor allocated in country j relative to country i falls in the tax rate in country j relative to country i . In the model, we make the simplifying assumption that the share of the fixed factor of a multinational in a specific country depends on the statutory tax rate in that country, relative to the weighted EU average (where we abstract from the value-added terms). The responsiveness of the fixed factor to this tax differential is set so as the replicate empirical estimates on the impact of corporate taxes on FDI.

1.3 Government

Tax bases regarding dividends and capital gains are aggregated over the firm types as:

$$Div(i, j) = Div^d(i, j) + Div^m(i, j) = (\hat{r}_{we} - g_y) E(i, j) \quad (1.99)$$

$$\Delta V(i, j) = \Delta V^d(i, j) + \Delta V^m(i, j) = g_y E(i, j) \quad (1.100)$$

with $E(i, j) = e(i, j)N^o(i)$. Corporate tax revenues are:

$$CIT(i) = \tau_\pi(i) \hat{\Pi}^d(i) + \tau_\pi(i) \left[\hat{\Pi}^m(i) + \sum_{j \neq i} \hat{\Pi}^f(j, i) \right]$$

Government consumption is assumed a fixed fraction ω_g of GDP (defined in 1.89). The government debt D^g is assumed to be a fixed fraction of GDP: $D^g(i) = \bar{d}_g(i)Y(i)$. The issue of new debt due to economic growth covers the deficit of the government:

$$\begin{aligned} & g_y \bar{d}_g(i)Y(i) + \tau_l(i)w(i)L(i) + \tau_c(i)[C^y(i) + C^o(i)] + CIT(i) \\ & \tau_d(i) \sum_j Div(i, j) + \tau_g(i) \sum_j \Delta V(i, j) + \tau_b(i) \sum_j \hat{r}_{wb} B(i, j) \\ & = \omega_g(i)Y(i) + tr^y(i)N^y(i) + tr^o(i)N^o(i) + \hat{r}_{wb} \bar{d}_g(i)Y(i) \end{aligned}$$

In the following we will express variables in per capita terms (denoted by lower case symbols), using as general rule:

$$x(i, j) = \frac{X(i, j)}{N^y(i)}$$

where the denominator refers to the population in the country of origin.¹⁶ The government budget becomes:

$$\begin{aligned} & \tau_l(i)w(i)l(i)N^y(i) + \tau_c(i)[c^y(i)N^y(i) + c^o(i)N^o(i)] + cit(i) \\ & [\tau_d(i)(\hat{r}_{we} - g_y)e(i) + \tau_g(i)g_y e(i) + \tau_b(i)\hat{r}_{wb}b(i)]N^o(i) \\ & = \omega_g(i)y(i)N^y(i) + tr^y(i)N^y(i) + tr^o(i)N^o(i) + (\hat{r}_{wb} - g_y)\bar{d}_g(i)y(i)N^y(i) \end{aligned} \quad (1.101)$$

where $cit(i)$ is the corporate tax revenues per capita (note that $\pi^f(j, i)$ is expressed per capita of country j).

1.4 Market Equilibria

1.4.1 Good markets

The total capital stock in country i is obtained by taking the sum over all active firms:

$$K(i) \equiv K^d(i) + K^m(i) + \sum_{j \neq i} K^f(j, i)$$

The sum of the financial distress costs is abbreviated as

$$c_b(i)K(i) \equiv c_b^d(i)K^d(i) + c_b^m(i)K^m(i) + \sum_{j \neq i} c_b^f(j, i)K^f(j, i) \quad (1.102)$$

Equilibrium on the goods market in each country requires (including a time subscript):

$$\begin{aligned} Y_t(i) &= C_t^y(i) + C_t^o(i) + K_{t+1}(i) - (1 - \delta_k - c_b(i))K_t(i) + \sum_{j \neq i} (1 + c_q(i, j))Q(i, j) + \\ & \omega_g(i)Y_t(i) + EX_t(i) + (s(i) - b(i) - e(i))N_t^y(i) \end{aligned}$$

where EX denotes total net exports of the final good, i.e. exclusive of Q (note that gross bilateral exports are undetermined). The last term at the right-hand side represents the resources which

¹⁶ Total pure profits per capita are defined as $\pi \equiv \pi^d + \pi^m + \sum_{j \neq i} \pi^f(j)$. Note that π^o in (1.8) is now rewritten as $\pi^o = \frac{\Pi}{N^o} = \frac{\Pi}{N^y} \frac{N^y}{N^o} = \pi(1 + g_n)^T$.

are lost in making the saving composite. In per capita terms the steady state equation for good market equilibrium, noting that both population and productivity grow, becomes:

$$y(i) = c^y(i) + \frac{c^o(i)}{(1+g_n)^T} + k(i)(1+g_y) - (1-\delta_k - c_b(i))k(i) + \sum_{j \neq i} (1+c_q(i,j))q(i,j) + \omega_g(i)y(i) + ex(i) + s(i) - b(i) - e(i) \quad (1.103)$$

1.4.2 Factor markets

Since domestic and foreign assets are assumed perfect substitutes, net foreign holdings of bonds (b_w) and equities (e_w) follow from equilibrium on each asset market (Notice that $N_{t-1}^y = N_t^o$):

$$\begin{aligned} b(i)N^o(i) + b_w(i)N^y(i) &= d_b^d(i)K^d(i) + d_b^m(i)K^m(i) + \sum_{j \neq i} d_b^f(j,i)K^f(j,i) + \bar{d}_g(i)Y(i) \\ e(i)N^o(i) + e_w(i)N^y(i) &= V^d(i) + V^m(i) \end{aligned} \quad (1.104)$$

When a country wants to issue more bonds than it holds, foreigners are willing to hold the excess amount ($b_w > 0$) at the given world interest rate. Analogously, domestic residents own part of the foreign firms when $e_w < 0$. Labour supply should equal total demand for labour, or:

$$l^d(i) + l^m(i) + \sum_{j \neq i} l^f(j,i)\omega_n(j,i) = l(i) \quad (1.106)$$

Extension: In the basic version the world interest rates are exogenous. In an extended version, the interest rates on bonds and equity are endogenized by postulating a simple reduced form. For each asset, a linear relation between the world interest rate and net capital demand of the EU is specified:

$$\hat{r}_{wx} = \gamma_x \frac{\sum_w x_w(i)N^y(i)}{\sum_y y(i)N^y(i)} + \gamma_x, \quad x = b, e \quad (1.107)$$

1.4.3 Balance of Payments

Net foreign assets are defined as the value of the assets a country owns minus the total value of all assets issued by that country:

$$\begin{aligned} FA(i) &= [B(i) + E(i)] - \bar{d}_g(i)Y(i) \\ &\quad - \left[V^d(i) + d_b^d(i)K^d(i) + V^{mm}(i) + d_b^m(i)K^m(i) + \sum_{j \neq i} \left(V^{mf}(j,i) + d_b^f(j,i)K^f(j,i) \right) \right] \end{aligned}$$

Using equilibrium on the capital market in (1.104) and (1.105), one can derive an alternative expression for the net foreign assets:

$$FA(i) = -[B_w(i) + E_w(i)] + \sum_{j \neq i} [V^{mf}(i,j) - V^{mf}(j,i)]$$

The Current Account equals the Trade Balance plus net foreign earnings on bonds, equities and FDI:

$$\begin{aligned} CA(i) &= -\hat{r}_{wb}B_w(i) - \hat{r}_{we}E_w(i) \\ &\quad + \sum_{j \neq i} \left[\hat{r}_{we}V^{mf}(i,j) + \Pi^f(i,j) - \hat{r}_{we}V^{mf}(j,i) - \Pi^f(j,i) \right] \\ &\quad + EX(i) + \sum_{j \neq i} [p_q(i,j)Q(i,j) - p_q(j,i)Q(j,i)] \end{aligned}$$

In view of the Balance of Payments definition $FA_{t+1} = (1 + g_y)FA_t = CA_t + FA_t$ one gets:

$$\begin{aligned}
& -(\hat{r}_{wb} - g_y)B_w(i) - (\hat{r}_{we} - g_y)E_w(i) + \\
& \sum_{j \neq i} \left[(\hat{r}_{we} - g_y)V^{mf}(i, j) + \Pi^f(i, j) - (\hat{r}_{we} - g_y)V^{mf}(j, i) - \Pi^f(j, i) \right] + \\
& EX(i) + \sum_{j \neq i} [p_q(i, j)Q(i, j) - p_q(j, i)Q(j, i)] = 0
\end{aligned}$$

The per capita expression is easily obtained:

$$\begin{aligned}
& -(\hat{r}_{wb} - g_y)b_w(i) - (\hat{r}_{we} - g_y)e_w(i) + \\
& \sum_{j \neq i} \left[(\hat{r}_{we} - g_y)v^{mf}(i, j) + \pi^f(i, j) + p_q(i, j)q(i, j) \right] - \\
& \sum_{j \neq i} \left[(\hat{r}_{we} - g_y)v^{mf}(j, i) + \pi^f(j, i) + p_q(j, i)q(j, i) \right] \omega_n(j, i) + ex(i) = 0 \quad (1.108)
\end{aligned}$$

where $\omega_n(j, i) \equiv N^y(j)/N^y(i)$ is a short-cut for the relative population sizes.

1.5 Solution method

The model is implemented in GAMS.¹⁷ It is solved as a Constrained Nonlinear System, for which the number of equations has to equal the number of variables.¹⁸ The price of the good is taken as the numeraire. Due to Walras law, one of the equations is redundant. In the GAMS-program the balance of payments condition (1.108) is dropped but checked afterwards.

¹⁷ Knowledge of the brief GAMS tutorial is sufficient for understanding the computer program.

¹⁸ Technical documentation can be found in www.gams.com/docs/pdf/cns.pdf or in www.gams.com/solvers/conopt.pdf (Appendix A13.2). This method does not allow that variables are at their bounds in the solution.

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